**Solution of Week 5.**

**Problem1.**

**There are N risky assets with returns .**

**The expected return and the variance-covariance are denoted by**

**Let be the weights of the risky assets in the portfolio.**

**Using the Lagrange Multiplier Methods, Find the optimal weights of portfolio under given the target expected return**

Let’s find the optimal portfolio with given expected return under the constraint .

For , we define the subject function that

Then, the solution satisfies the followings:

By inserting in the above constraints,

Therefore, the optimal weights of the portfolio are

, , ,

참고:

행렬 편미분 공식<https://m.blog.naver.com/enewltlr/220918689039>

**Problem 2.**

**There are N risky assets with returns and the risk-free asset with risk-free rate .**

**The expected return and the variance-covariance are denoted by**

**Let be the weights of the risky assets in the portfolio. The weight of the risk-free asset is**

**Solve the following sub-problems.**

**1) Using the Lagrange Multiplier Methods, Find the optimal weights of portfolio under given the target expected return**

Let’s find the optimal portfolio with given expected return under the constraint .

For , we define the subject function that

Then, the solution satisfies the followings:

By inserting in the above constraint,

Therefore, the optimal weights under the given expected return are

**2) Find the weight vector of the Tangency Portfolio.**

Let be the weight of the tangency portfolio. Then,

The tangency portfolio exists on the Efficient frontier, so it consists of only risky assets. Therefore, .

we have which gives .

**3) Find the variance of the optimal portfolio.**

We have obtained the optimal portfolio with expected return as follows:

The variance of optimal portfolio is

**4) Use the risk-aversion parameter to indicate the weight of the optimal portfolio. (Definition of risk aversion parameter )**

The risk aversion parameter is

Therefore, the weights of optimal portfolio with risk aversion parameter are

**5) Show that under a given constraint, all optimal portfolios have the same Sharpe Ratio and that all optimal portfolios are above the CAL of the Tangency portfolio.**

Note that

Then, for any target expected return the optimal weights of portfolio are

The definition of Sharpe Ratio (slope of the risk-return plane) is

By the result of sub-problem 3

Because the ratio of risk to return for any optimal portfolio is constant,

Therefore, the Capital Asset Line (CAL) of the tangency portfolio is

**Problem 3.**

**Explain that Tangency Portfolio becomes Market Portfolio under the assumption of CAPM, and show that CAL of Tangency Portfolio becomes CML (Capital Market Line).**

From the one-fund theorem every individual will form a portfolio that is a mix of the risk-free asset and the single, risky one fund. Thus, the one fund in the theorem is really the only fund that is used and must equal the market portfolio. Investors solve the mean-variance portfolio problem using their common estimates, and they place orders in the market to acquire their portfolios. If the orders placed do not match what is available, the prices must change. The prices of assets under heavy demand will increase; the prices of assets under light demand will decrease. In other words, prices adjust to drive the market to efficiency. These price changes affect the estimates of asset returns directly, and hence investors will recalculate their optimal portfolios. This process continues until demand exactly matches supply; that is, it continues until there is equilibrium. After other people have made the adjustments, we can be sure that the efficient portfolio is the market portfolio.

**Problem 4.**

1. **Follow the proof on page 184 of the textbook to induce the formula of CAPM.**

The tangency condition can be translated into the condition that the slope of the curve is equal to the slope of the capital market line at the point $. To set up this condition we need to calculate a few derivatives.

thus, and this slope must equal the slope of the capital market line. Hence,

solving for , obtaining the final result

**2) Derive the CAPM formula for by using Equation (6.9) in Chapter 6 of the Text book.**

**Hint: Note that Apply equation (6.9) both to asset k and to the market itself.**

Start with Markowitz Mean-Variance optimization,

The optimal portfolio is the market portfolio by the assumption of CAPM. So,

The risk aversion parameter is

Therefore,

**Problem 5.**

**Is the beta of securities a constant? Write down your inference.**

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